Part I

Reading Assignments
Part II

Introduction
We have often mentioned growth rates
We have often mentioned growth rates

- How are they calculated?
Introduction

We have often mentioned growth rates

- How are they calculated?
- How can we look at or analyze growth rates across countries and regions?
Part III

Introduction
Problem

Suppose your company expects the following revenue stream from a new product. What is the average growth rate for the revenue?

<table>
<thead>
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<tr>
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<td>$400</td>
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<tr>
<td>2</td>
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<td>$675</td>
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Problem
Suppose your company expects the following revenue stream from a new product. What is the average growth rate for the revenue?

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Calculate period-over-period growth using simple return formula:
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\[ 25.00\% = \left( \frac{\$500}{\$400} - 1 \right) \times 100 \]
Calculate period-over-period growth using simple return formula:

<table>
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<tr>
<th></th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>25.00%</td>
<td>20.00%</td>
<td>12.50%</td>
<td>3.70%</td>
<td></td>
</tr>
</tbody>
</table>

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Averaging the growth rates using a simple STAT 101 arithmetic averaging (i.e., calculating $\bar{X}$) yields
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$$\text{Average} = \frac{1}{4} (0.25 + 0.20 + 0.125 + 0.037)$$

$$= 0.153$$
Averaging the growth rates using a simple STAT 101 arithmetic averaging (i.e., calculating $\bar{X}$) yields

$$\text{Average} = \frac{1}{4} (0.25 + 0.20 + 0.125 + 0.037) = 0.153$$

or 15.3% average growth
Now grow the beginning revenue by the average growth rate of 1.153 for 5 periods
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<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>$461</td>
</tr>
<tr>
<td>2</td>
<td>$532</td>
</tr>
<tr>
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<td>$613</td>
</tr>
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<td>$707</td>
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Now grow the beginning revenue by the average growth rate of 1.153 for 5 periods

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Notice that the last value overstates the true last period revenue.
The problem is that $\bar{X}$ is good for problems involving quantities that naturally add since

$$A = \sum X_t$$
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$$n\bar{X}_A = \sum X_t$$
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- The arithmetic average, $\bar{X}_A$, is representative of the data
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- The arithmetic average, $\bar{X}_A$, is representative of the data
- It reproduces the sum exactly
The problem is that \( \bar{X} \) is good for problems involving quantities that naturally add since

\[
n \bar{X}_A = \sum X_t
\]

- The **arithmetic** average, \( \bar{X}_A \), is representative of the data
- It reproduces the **sum** exactly
- But not in this case!!
A better average is the **geometric** average which reproduces a product of terms
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- It is applicable when multiplication is the natural operation
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- It is applicable when multiplication is the natural operation.
A better average is the geometric average which reproduces a product of terms.

- It is applicable when multiplication is the natural operation

$$\bar{X}_G = \left( \prod_{t=1}^{n} X_t \right)^{\frac{1}{n}}$$

where \( n \) is the number of terms.
Note that...
Note that...

\[ X_n = X_0 (1 + g)^n \]

where \( n = 4 \) in our problem, the number for the last period, even though there are 5 numbers. Clearly, \( n \) is one less than the number of data points. The fifth data point is the one for period 0, today.
Note that...

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But...

\[
\frac{X_n}{X_0} = \frac{X_n}{X_{n-1}} \cdot \frac{X_{n-1}}{X_{n-2}} \cdots \frac{X_2}{X_1} \cdot \frac{X_1}{X_0} = (1 + g)^n
\]
Note that...

\[ X_n = X_0 (1 + g)^n \]

where \( n = 4 \) in our problem, the number for the last period, even though there are 5 numbers. Clearly, \( n \) is one less than the number of data points. The fifth data point is the one for period 0, today.

But...

\[ \frac{X_n}{X_0} = \frac{X_n}{X_{n-1}} \frac{X_{n-1}}{X_{n-2}} \ldots \frac{X_2}{X_1} \frac{X_1}{X_0} = (1 + g)^n \]

\[ \prod_{i=1}^{n} RATIO_i = (1 + g)^n \]
Clearly, this operation just involves a product, not a sum

- The arithmetic average would be inappropriate since it works on sums
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- The geometric average is superior
Clearly, this operation just involves a product, not a sum

- The arithmetic average would be inappropriate since it works on sums
- The geometric average is superior
  - It preserves the ratio nature of the data
We can solve for $g$ as...
We can solve for $g$ as... 

$$g = \left( \prod_{i=1}^{n} RATIO_i \right)^{\frac{1}{n}} - 1$$
We can show that, in general, ...

\[ \frac{1}{n} \sum x \geq (\prod x) \frac{1}{n} \]
We can show that, in general, ...

$$\frac{1}{n} \sum x \geq (\prod x)^{\frac{1}{n}}$$

so we almost always overstate true average.
For our problem, use…
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\[ \bar{X}_G = \left( \prod_{i=1}^{4} RATIO_i \right)^{\frac{1}{4}} - 1 \]
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\[ \bar{X}_G = \left( \prod_{i=1}^{4} RATIO_i \right)^{\frac{1}{4}} - 1 \]

\[ = \left( \frac{700}{400} \right)^{\frac{1}{4}} - 1 \]
For our problem, use...

\[ \bar{X}_G = \left( \prod_{i=1}^{4} RATIO_i \right)^{\frac{1}{4}} - 1 \]

\[ = \left( \frac{700}{400} \right)^{\frac{1}{4}} - 1 \]

\[ = 0.1502 \]
For our problem, use...

\[
\bar{X}_G = \left( \prod_{i=1}^{4} RATIO_i \right)^{\frac{1}{4}} - 1
\]

\[
= \left( \frac{700}{400} \right)^{\frac{1}{4}} - 1
\]

\[= 0.1502\]

of 15.02% average annual growth.
Now grow revenues again using 1.1502
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Now grow revenues again using 1.1502

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Notice that the last value is exact.
Part IV

Key Statistical Tools
Since a country database sometimes has multiple regions and countries within the regions, a good tool is a boxplot.
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- Boxplots are based on fundamental statistics involving percentiles.
Since a country database sometimes has multiple regions and countries within the regions, a good tool is a boxplot.

- Boxplots are based on fundamental statistics involving percentiles.
- The percentiles allow better analysis when there are extreme values (outliers) that would distort conventional statistics (e.g., means, standard deviations).
### Common Statistics Based on Percentiles

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^{th}$</td>
<td>Minimum</td>
</tr>
<tr>
<td>$25^{th}$</td>
<td>First Quartile</td>
</tr>
<tr>
<td>$50^{th}$</td>
<td>Median</td>
</tr>
<tr>
<td>$75^{th}$</td>
<td>Third Quartile</td>
</tr>
<tr>
<td>$100^{th}$</td>
<td>Maximum</td>
</tr>
</tbody>
</table>
Definition
Quartiles Q1 and Q3 are numerical values that divide a sample of observations into groups based on “25” such that...
- One-fourth (25%) of the data are less than Q1
- Three-fourths (75%) are less than Q3
Ranges

We can calculate the spread in the data from the quartiles...

Definitions

Conventional range: \( R = \text{Maximum} - \text{Minimum} \)

Interquartile range: \( IQR = Q_3 - Q_1 \)
We can summarize the percentile-based statistics with five number summary...
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1. **Minimum**
We can summarize the percentile-based statistics with five number summary...

1. **Minimum**
2. **1st Quartile (Q1)**
We can summarize the percentile-based statistics with five number summary…

1. Minimum
2. 1st Quartile (Q1)
3. Median

Five-Number Summary

1. Minimum
2. 1st Quartile (Q1)
3. Median
We can summarize the percentile-based statistics with five number summary...

1. Minimum
2. 1st Quartile (Q1)
3. Median
4. 3rd Quartile (Q3)
We can summarize the percentile-based statistics with five number summary…

1. **Minimum**
2. **1\textsuperscript{st} Quartile (Q1)**
3. **Median**
4. **3\textsuperscript{rd} Quartile (Q3)**
5. **Maximum**
A five-number summary allows you to easily calculate...
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- **Ranges**
A five-number summary allows you to easily calculate...

- **Ranges**
  - *Conventional* = $\text{Max} - \text{Min}$
A five-number summary allows you to easily calculate. . .

- **Ranges**
  - Conventional = $Max - Min$
  - Interquartile = $Q3 - Q1$
A five-number summary allows you to easily calculate...

- **Ranges**
  - Conventional = $Max - Min$
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- **Skewness**
A five-number summary allows you to easily calculate... 

- **Ranges**
  - Conventional = $Max - Min$
  - Interquartile = $Q3 - Q1$

- **Skewness**
  - Skewed right: $Q3 - Median > Median - Q1$
  - Skewed left: $Median - Q1 > Q3 - Median$

- Outliers
  - Greater than $Q3 + 1.5 \times IQR$
  - Less than $Q1 - 1.5 \times IQR$
A five-number summary allows you to easily calculate...

- **Ranges**
  - Conventional = $Max - Min$
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- **Outliers**
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  - Skewed left: $\text{Median} - Q1 > Q3 - \text{Median}$

- **Outliers**
  - Greater than $Q3 + 1.5 \times IQR$
  - Less than $Q1 - 1.5 \times IQR$
### Five-Number Summary

<table>
<thead>
<tr>
<th>Variable</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grades</td>
<td>68</td>
<td>73.5</td>
<td>81</td>
<td>87.25</td>
<td>91</td>
</tr>
</tbody>
</table>
Example Boxplot

Variable Values

Median

Lower Quantile (25%)

Upper Quantile (75%)

Outlier

Randomly Generated Data

Length of box shows skewness and dispersion of bulk of the data
Length of tails also shows skewness
Desired shape: Symmetric box and equally long tails
Boxplots are very versatile
Boxplots are very versatile

- **Reveal...**
Boxplots are very versatile

- Reveal...
  - Distribution of data
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness

- Look for...
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness

- Look for...
  - Symmetry
Boxplots are very versatile

- Reveal...
  - Distribution of data
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- Reveal...
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- Look for...
  - Symmetry
  - Outliers
  - Elongated tails
Boxplots are very versatile

- Reveal...
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- Look for...
  - Symmetry
  - Outliers
  - Elongated tails

- Normal distribution pattern...
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness

- Look for...
  - Symmetry
  - Outliers
  - Elongated tails

- Normal distribution pattern...
  - Symmetric box
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness

- Look for...
  - Symmetry
  - Outliers
  - Elongated tails

- Normal distribution pattern...
  - Symmetric box
  - Even tails
Boxplots are very versatile

- Reveal...
  - Distribution of data
  - Outliers and skewness

- Look for...
  - Symmetry
  - Outliers
  - Elongated tails

- Normal distribution pattern...
  - Symmetric box
  - Even tails
  - No outliers
FIGURE 1.2 Estimates of the distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.
FIGURE 1.8
The evolution of income per capita in the United States, the United Kingdom, Spain, Singapore, Brazil, Guatemala, South Korea, Botswana, Nigeria, and India, 1960-2000.
We can look at some data to further understand how countries and regions have grown.
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*Data Source: Penn World Trade Tables (v. 5.6)*
We can look at some data to further understand how countries and regions have grown

- **Data Source:** Penn World Trade Tables (v. 5.6)
- **Assembled by Summers and Heston**
Some Data

We can look at some data to further understand how countries and regions have grown

- **Data Source:** Penn World Trade Tables (v. 5.6)
- **Assembled by Summers and Heston**
- **Contains data on 152 countries divided into 6 regions (Africa, Central-North America, Asia, Europe, Oceania, South America)**
Some Data

(Continued)

Penn Data problems...
Penn Data problems...

- Not all countries have complete data
Penn Data problems. . .

- Not all countries have complete data
- Population, RGDP, capital stock, and standard of living are missing for a number of years for each country
Penn Data problems... 

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- Western developed nations have very complete data, as should be expected
Some Data (Continued)

Penn Data problems...

- Not all countries have complete data
- Population, RGDP, capital stock, and standard of living are missing for a number of years for each country
- Western developed nations have very complete data, as should be expected
- **Timeliness is an issue**
Distribution of Real GDP per Worker Growth Rates by Major Geographic Regions

Average Annual Real GDP per Worker Growth Rates:
- Africa: -8% (n = 39)
- Central-North America: -6% (n = 13)
- Asia: -4% (n = 25)
- Europe: -2% (n = 22)
- Oceania: 0% (n = 3)
- South America: 2% (n = 12)

Source: Penn World Tables, v5.6
Data: Real GDP per Worker
Years vary
n = 114 countries

USA Growth for 1950 to 1990: 1.5% per Year
Median Worldwide Growth (Horizontal Line): 2.1% per Year

Intro to Africa
Real GDP per Worker Growth Rates vs. Initial Level of RGDP per Worker

Source: Penn World Tables, v5.6
Data: Real GDP per Worker
1950-1990
n = 46 countries
Distribution of Population Growth Rates by Major Geographic Regions

Average Annual Population Growth Rates

<table>
<thead>
<tr>
<th>Region</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>0%</td>
</tr>
<tr>
<td>Central-North America</td>
<td>2%</td>
</tr>
<tr>
<td>Asia</td>
<td>4%</td>
</tr>
<tr>
<td>Europe</td>
<td>6%</td>
</tr>
<tr>
<td>Oceania</td>
<td>8%</td>
</tr>
<tr>
<td>South America</td>
<td>10%</td>
</tr>
</tbody>
</table>

Source: Penn World Tables, v5.6
Years Vary
n = 147 countries

USA Growth for 1950 to 1992: 1.2% per Year
Median Worldwide Growth (Horizontal Line): 2.4% per Year
Real GDP per Worker Growth Rates
vs. Population Growth Rates

Average Annual Population Growth Rates (%)

Average Annual Real GDP per Worker Growth Rates (%)

Source: Penn World Tables, v5.6
Data: Real GDP per Worker
1950-1990
n = 46 countries
Line is LOWESS Smooth showing trend
Call: lm(formula = RGDPGrowth ~ POPGrowth, data = x)

Residuals:
  Min     1Q  Median     3Q  Max
-3.528 -0.7335 0.05769 0.8061 2.753

Coefficients:
                Value  Std. Error  t-Pr(>|t|)  Pr(>|t|)
(Intercept)   3.4961  0.3565  9.8053 0.0000
POPGrowth  -0.7004  0.1741 -4.0233 0.0002

Residual standard error: 1.166 on 44 degrees of freedom
Multiple R-Squared:  0.2689
F-statistic: 16.19 on 1 and 44 degrees of freedom, the p-value is 0.0002224

Correlation of Coefficients:
  (Intercept)
POPGrowth -0.8762

Elasticity: -0.56
Distribution of Standard of Living Growth Rates by Major Geographic Regions

Average Annual Standard of Living Growth Rates

- Africa: -4%
- Central-North America: -2%
- Asia: 0%
- Europe: 2%
- Oceania: 4%
- South America: 6%

Source: Penn World Tables, v5.6
Years vary
n = 89 countries

USA Growth for 1970 to 1989: -0.1% per Year
Median Worldwide Growth (Horizontal Line): 0% per Year

Standard of Living is real personal income per capita
Real GDP per Worker Growth Rates vs. Capital Stock per Worker Growth Rates

Average Annual Capital Stock per Worker Growth Rates (%)

Source: Penn World Tables, v5.6
Data: Real GDP per Worker and Capital Stock per Worker
1965-1990
n = 47 countries
Line is LOWESS Smooth showing trend
Call: `lm(formula = x$RGDPGrowth ~ x$KAPWGrowth)`

Residuals:

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.659</td>
<td>-0.6385</td>
<td>-0.09679</td>
<td>0.7076</td>
<td>3.577</td>
</tr>
</tbody>
</table>

Coefficients:

|                      | Value   | Std. Error | t- Value | Pr(>|t|) |
|----------------------|---------|------------|----------|----------|
| (Intercept)          | 0.4485  | 0.3163     | 1.4181   | 0.1630   |
| x$KAPWGrowth         | 0.4343  | 0.0774     | 5.6128   | 0.0000   |

Residual standard error: 1.196 on 45 degrees of freedom

**Multiple R-Squared: 0.4118**

F-statistic: 31.5 on 1 and 45 degrees of freedom, the p-value is 1.169e-006

Correlation of Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>(Intercept)</th>
<th>x$KAPWGrowth</th>
</tr>
</thead>
<tbody>
<tr>
<td>x$KAPWGrowth</td>
<td>-0.834</td>
<td></td>
</tr>
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</table>

**Elasticity:** 0.24
Population Growth Rates
Median of Rates

Year-over-Year Growth, 1966-1990
Output-Capital Ratio Median of Rates

Note: Oceania is one country

Period, 1965-1990